Magnetohydrodynamics of Planets and Stars

What Idealised Numerical Simulations and Laboratory Experiments Can Teach Us

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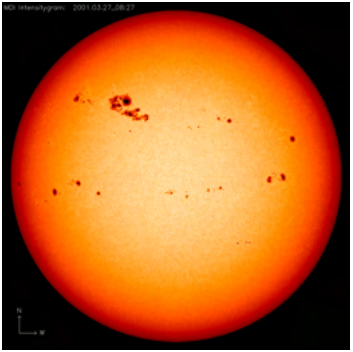
Centre for Fluid and Complex Systems, Coventry University

EMFL User Meeting, 11th June 2024

Sunspots and the Solar Dynamo



- sunspots observ
  since 800 BC
- thought to be sc
  analogues of te
  volcanoes or to

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Wöhler ist eine ausführlichere Mittheilung über das ganze Phänomen und die chemisch-mineralogische Constitution dieser Meteoriten in der Kürze zu erwarten. *P.*

XVI: *Ueber die Sonnenflecke; aus einem Briefe an Hrn. A. v. Humboldt von Ch. H. F. Peters.*

Cambridge, b. Boston 11. Sept. 1855.

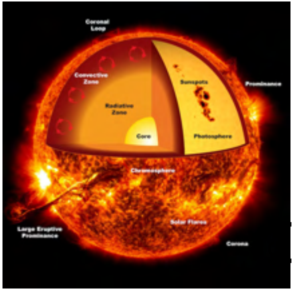
— Vielleicht darf ich es wagen, einige Hauptpunkte von der kleinen Arbeit herzusetzen, mit welcher ich mich hier habilitirte. Im J. 1845 begann ich in Neapel eine Reihe von Messungen aller Sonnenflecke, besonders in der Absicht, zu erforschen, ob auf der Sonnenoberfläche gewisse feste Localitäten existiren, wo diese Flecke vorzugsweise entstehen. Bis Ende 1846, wo eine Unterbrechung in der Reihe eintrat, hatte ich 813 Oerter von 286 Flecken bestimmt, und die Discussion der daraus hergeleiteten heliocentrischen Coordinaten führt zu folgenden Resultaten: 1) Die Flecke sind nicht fest, sondern alle haben eine Bewegung gegen den Aequator. 2) Wenn ein Fleck in der Nähe eines anderen ausbricht, so bewegt sich letzterer nach der vom ersteren abgwandten Seite. 3) Die Flecke haben ebenfalls eine Eigenbewegung nach Westen; diefs ist jedoch kein direct erhaltenes Resultat, sondern beruht auf meiner Ansicht vom Ursprung der Sonnenflecke, im Verein mit der Bemerkung, dafs in einem System ein neuer Fleck immer an der Ostseite entsteht. 4) Gewisse Stellen sind vorzugsweise productiv an Flecken: in heliocentr. Breite die zwei Zonen um 21° N und 17° S, in Länge, wenigstens in d. J. 1845 und 1846, vier Stellen. Die Erscheinungen des Entstehens und Wachsthums der Flecke, die der Fackeln, und die obigen Thatsachen, lassen sich bis ins Detail vereinigen, wenn man annimmt, dafs auf dem Sonnenkörper Etwas wie Vulkane existirt, welche gasförmige Materien aussenden. —

Gedruckt bei A. W. Schade in Berlin, Grünstr. 18.

- ⇝ George E. Hale[†]: fine structures around sunspots resembling iron filings around a magnet
- ⇝ Zeeman splitting in the presence of a magnetic fie

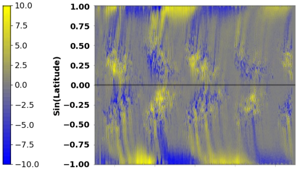
[*] PETERS, C.H.F. Ueber die Sonnenflecke. *Annalen der Physik* (1855)
[†] HALE, G.E. On the Probable Existence of a Magnetic Field in Sun-Spots. *Astrophysical Journal* (1908)

Sunspots and the Solar Dynamo

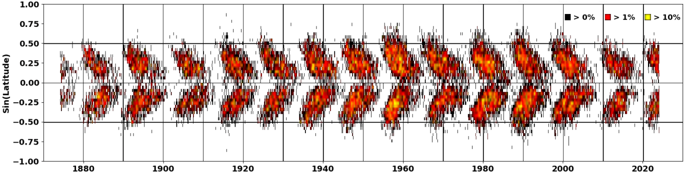


- magnetic forces responsible for nearly all activity and variability of the Sun
- Hale cycle - magnetic field back to original polarity (22 years)
- Sir Larmor suggested solar dynamo*

Longitudinally Averaged Magnetic Field

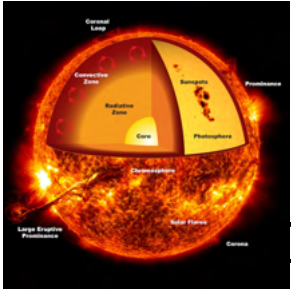


Sunspot Area Coverage



*Sir Larmor, J. How could a Rotating Body such as the Sun become a Magnet? *Rep. Brit. Assoc. Adv. Sci. A* (1919)
†Vasil, G.M. et al. The solar dynamo begins near the surface. *Nature* (2024)
‡Vasil, G.M., Julien, K. and Featherstone, N.A. Rotation suppresses giant-scale solar convection. *Proc. Nat. Acad. Sci.* (2021)

Sunspots and the Solar Dynamo



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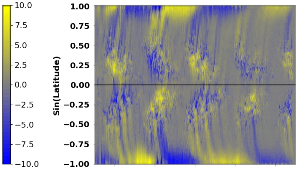
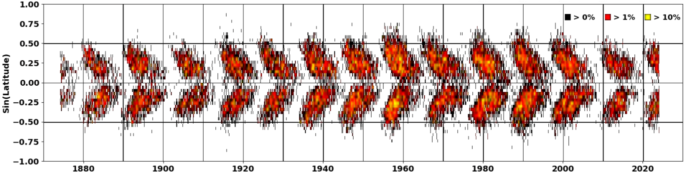


Sunspot Area Coverage



- ⇝ originating from tachocline or near-surface instability?†
- ⇝ rapidly rotating "underlayer"? ‡
- ⇝ why is corona 150 to 450 times hotter than surface?

⋮

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Planetary Dynamos

- planetary magnetic fields are generated via self-sustained dynamo action

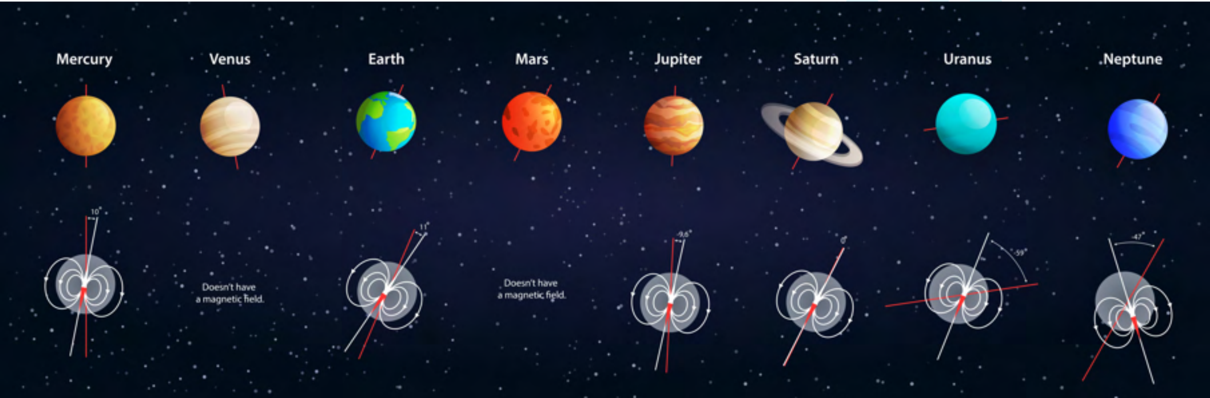
⇝ maintain magnetic field against decay through Ohmic dissipation

- **rotating turbulent convection in**

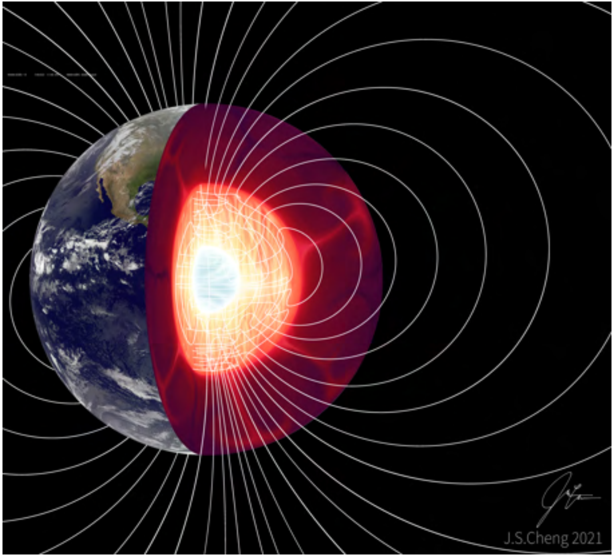
⇝ liquid metal cores of rocky planets

⇝ metallic hydrogen envelopes of gas giants

⇝ superionic ice layer of ice giants



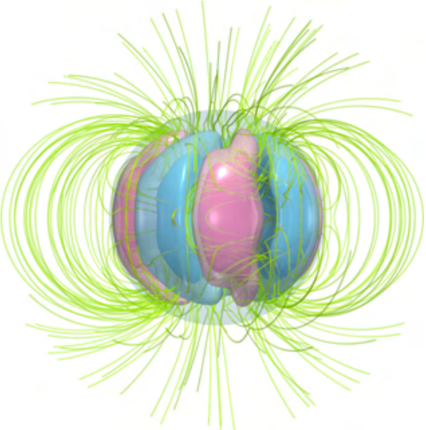
Geodynamo



- Earth's liquid metal core: remote, under-constrained systems
- crustal magnetism forms a magnetic curtain

$\rightsquigarrow$ only observe length scales larger than $\sim 1700\,\mathrm{km}$

Dynamo Models and Characteristic Length-Scales ℓ



$$\mathrm{Pr} = 1,\ \mathrm{Ra} = 10^5,\ \mathrm{Pm} = 5,\ \mathrm{Ek} = 10^{-3}$$

- numerical simulations easily generate dynamos[*]

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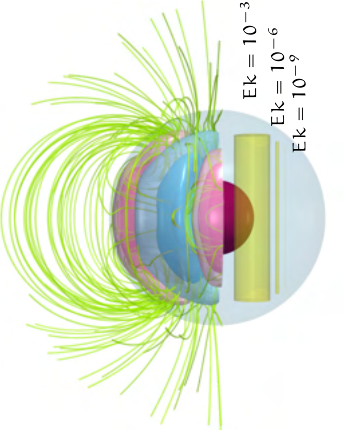
[*] MagIC, Wicht et al., *Astrophys. Source Code Lib.*, `https://magic-sph.github.io/` (2017)

[†] Aurnou & King, *Proc. R. Soc. A* 473 (2017)

Dynamo Models and Characteristic Length-Scales ℓ



$$\mathrm{Pr} = 1,\ \mathrm{Ra} = 10^5,\ \mathrm{Pm} = 5,\ \mathrm{Ek} = 10^{-3}$$

- ○ numerical simulations easily generate dynamos*
- ⇝ but only for unrealistic values of the control parameters, esp. Ekman numbers†

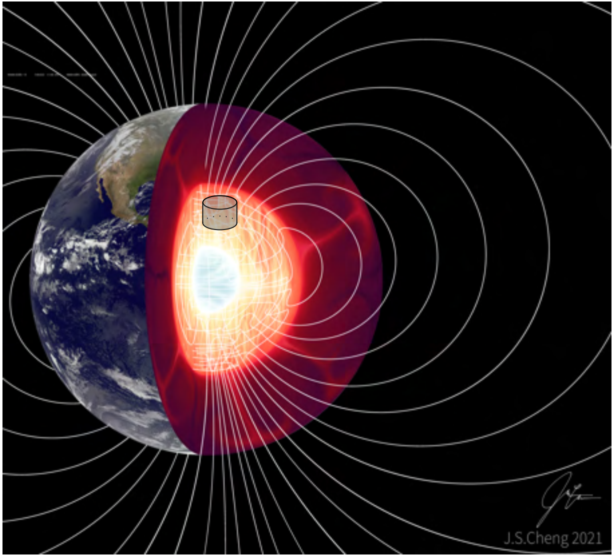
$$\mathrm{Ek} = \frac{\nu}{2\Omega H^2} = 10^{-3} \rightsquigarrow \ell \propto \mathrm{Ek}^{1/3} \sim 1000\,\mathrm{km}$$

- ⇝ extrapolation to Earth-like values:

$$\mathrm{Ek}_\oplus = 10^{-15} \rightsquigarrow \ell \sim 100\,\mathrm{m}$$

*MagIC, Wicht et al., *Astrophys. Source Code Lib.*, `https://magic-sph.github.io/` (2017)
†Aurnou & King, *Proc. R. Soc. A* 473 (2017)

Geodynamo



- Earth's liquid metal core: remote, under-constrained systems
- crustal magnetism forms a magnetic curtain

⇝ only observe length scales larger than ∼ 1700 km

⇝ objective: understanding the fundamentals of planetary core turbulence using **idealised models**

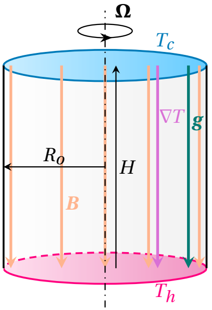
Sorting puzzle pieces into a picture. . .

- most important puzzle pieces:
  - turbulent convection (buoyancy)
  - rotation (Coriolis)
  - magnetic fields (Lorentz)

Sorting puzzle pieces into a picture. . .

- most important puzzle pieces:
  - turbulent convection (buoyancy)
  - rotation (Coriolis)
  - magnetic fields (Lorentz)
- methods of solution:
  - theoretical approaches
  - numerical simulations
  - laboratory experiments

From Physical Phenomena to Non-Dimensional Equations



- mass conservation:

$$\nabla \cdot \mathbf{u} = 0$$

- momentum conservation:

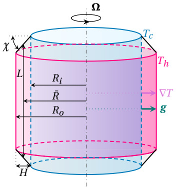
$$D_t \mathbf{u} = -\nabla p + \sqrt{\frac{\mathrm{Pr}}{\mathrm{Ra}\,\gamma^3}}\,\nabla^2 \mathbf{u} + T\hat{\mathbf{e}}_z - \sqrt{\frac{\mathrm{Pr}\gamma}{\mathrm{Ra}\,\mathrm{Ek}^2}}\,\hat{\mathbf{e}}_z \times \mathbf{u} + \sqrt{\frac{\mathrm{Ch}^2 \mathrm{Pr}\,\gamma}{\mathrm{Ra}}}\,(\mathbf{j} \times \hat{\mathbf{e}}_z)$$

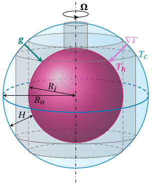
- entropy (temperature) equation:

$$D_t T = \sqrt{\frac{1}{\mathrm{Ra}\mathrm{Pr}\,\gamma^3}}\,\nabla^2 T$$

- induction equation (quasi-static low-Rm)

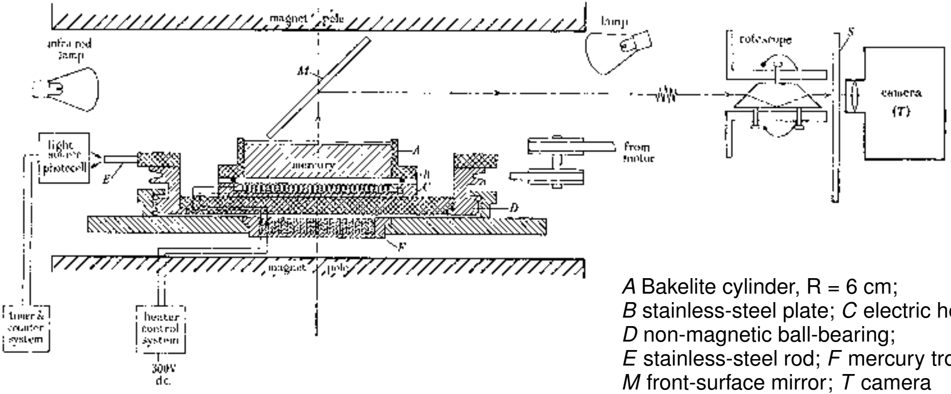
$$\left.\begin{aligned} \nabla \cdot \mathbf{j} &= 0 \\ \mathbf{j} &= -\nabla \Phi + (\mathbf{u} \times \hat{\mathbf{e}}_z) \end{aligned}\right\} \nabla^2 \Phi = \nabla \cdot (\mathbf{u} \times \hat{\mathbf{e}}_z)$$





- control parameters:

$$\mathrm{Ra} = \frac{\alpha g \Delta H^3}{\kappa \nu}, \quad \mathrm{Pr} = \frac{\nu}{\kappa}, \quad \gamma = \frac{R}{H}, \quad \mathrm{Ek} = \frac{\nu}{2\Omega H^2}, \quad \mathrm{Ch} = \frac{\sigma B_0^2 H^2}{\rho_0 \nu}$$



A Bakelite cylinder, R = 6 cm;
B stainless-steel plate; *C* electric heater;
D non-magnetic ball-bearing;
E stainless-steel rod; *F* mercury trough;
M front-surface mirror; *T* camera

FIGURE 1. A schematic diagram of experimental arrangement. *A*, Bakelite cylinder; *B*, stainless-steel plate; *C*, electric heater; *D*, non-magnetic ball-bearing; *E*, stainless-steel rod; *F*, mercury trough; *M*, front-surface mirror; *S*, rotary shutter; *T*, camera.

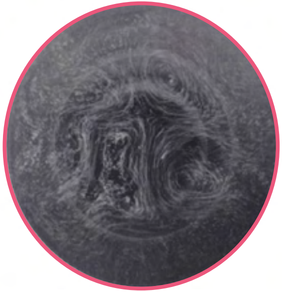
*NAKAGAWA Y., *Proc. Roy. Soc. A* (1959)

Nakagawa's Experiments in Liquid Mercury*

- ○ magnetic field strength increases from left to right, $B = \{125, 750, 1000, 3000\}$ Gs
- $\rightsquigarrow$ $Ch = \{9.47 \times 10^1, 3.47 \times 10^3, 6.17 \times 10^3, 5.54 \times 10^4\}$
- ○ constant rotation: $Ek = 1.2 \times 10^{-4}$
- ○ bright streaks represent pathlines created by using sand as free surface tracer particles

$\Lambda = 0.01, Ra \approx 1.3 \times 10^5$

$\Lambda = 0.41, Ra \approx 3.7 \times 10^5$

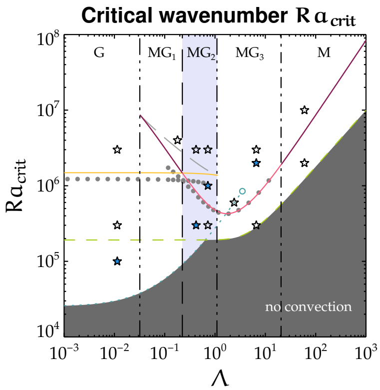


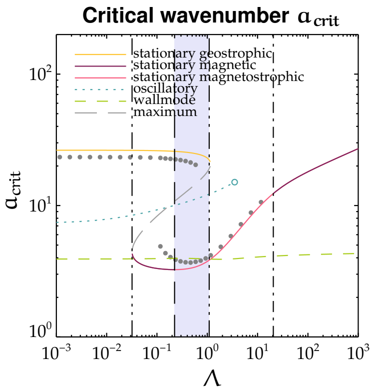
$\Lambda = 0.74, Ra \approx 3.4 \times 10^5$

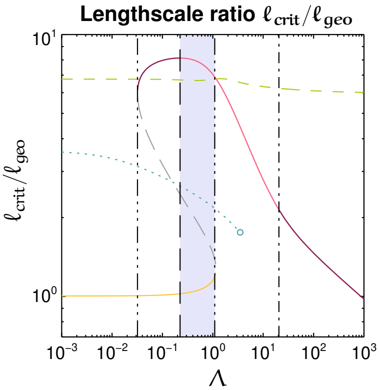
$\Lambda = 6.48, Ra \approx 8.2 \times 10^5$

*Nakagawa Y., *Proc. Roy. Soc. A* (1959)

Theoretical Linear Stability Predictions*







- liquid mercury, $\mathrm{Pr} = 0.025$, aspect ratio $\Gamma = 8$
- $\mathrm{Ek} = 1.2 \times 10^{-4}$ fixed, Ch/Λ varied, test different Ra at fixed $\Lambda = \mathrm{Ch}\,\mathrm{Ek}$

*Horn, S. & Aurnou, J. M. (2022). *Proc. Roy. Soc. A* 478; Murphy, J. O., & Steiner J. M. (1975). *Proc. Roy. Soc. Lond. A.* 347

Donna DeEtte Elbert (1928–2019)* †

- ○ started working with Chandrasekhar in 1948
- ○ no formal degree in mathematics (BFA in 1974)
- ⇝ 30-year collaboration, co-authored 16 papers
- ⇝ carried out almost all numerical computations
- ⇝ developed solutions more elegant than Chandrasekhar's original ones
- ⇝ **first to describe coexistence range of large-scale magnetostrophic and small-scale geostrophic modes**; footnote in *Hydrodynamic and Hydromagnetic Stability* (1961)

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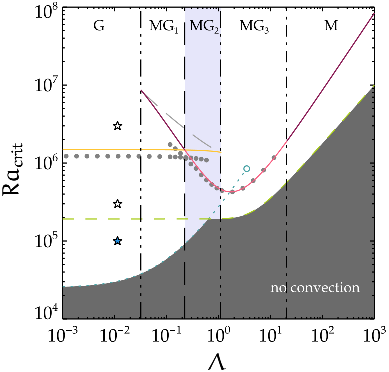
*photos courtesy of Dianne Hofner Saphiere, Susan Elbert Steele, Joanne Elbert Kantner

†Horn, S. & Aurnou, J.M. The Elbert range of magnetostrophic convection. I. Linear theory. *Proc. Roy. Soc. A* (2022)

Geostrophic Regime (G) - Nakagawa's 1st Case†

$\Lambda = 0.0114, \mathrm{Ch} = 9.5 \times 10^1, \mathrm{Ra} = 1 \times 10^5$



vertical velocity u_z, topview

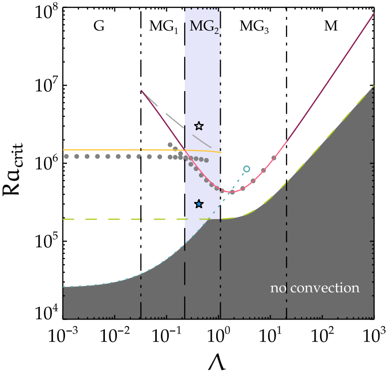
- no wallmodes, no stationary modes
- oscillatory: $Ra/Ra_o = 3.3$; $a_O = 7.921 \Rightarrow n \approx 17^*$

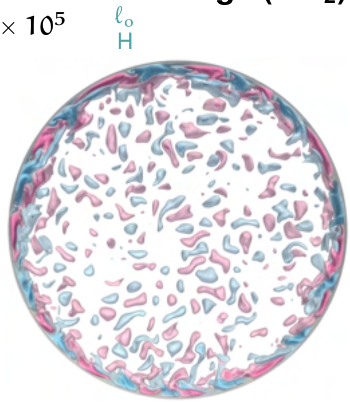
$^*\ell \approx \lambda_c/2 \Rightarrow n \approx \Gamma/\ell = \Gamma a_{crit}/\pi$

†HORN, S. & AURNOU, J.M.. The Elbert range of magnetostrophic convection. II. Comparing Linear Theory to Nonlinear Low-Rm Simulations; under review (2024)

Elbert's magnetostrophic coexistence range (MG_2) - Nakagawa's 2nd Case

$\Lambda = 0.42, \mathrm{Ch} = 3.5 \times 10^3, \mathrm{Ra} = 3 \times 10^5$



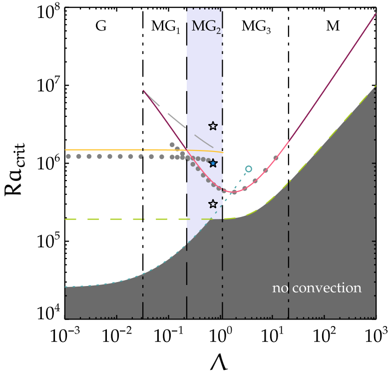


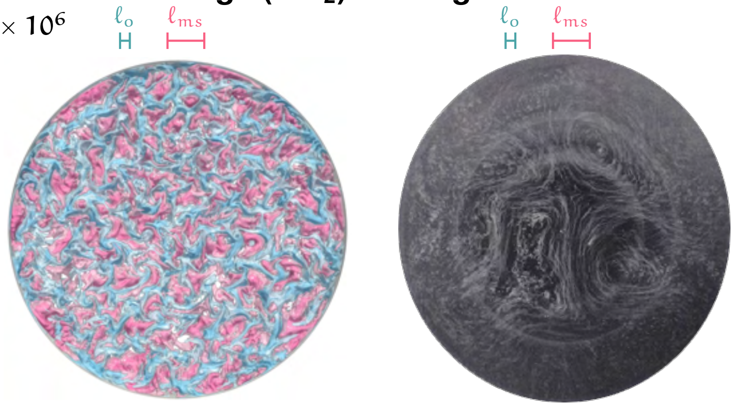
vertical velocity u_z, topview

- no stationary modes
- oscillatory: $\mathrm{Ra}/\mathrm{Ra}_o = 2.4$; $a_O = 10.750 \Rightarrow n \approx 27$
- wallmodes: $\mathrm{Ra}/\mathrm{Ra}_w = 1.6$, $a_w = 3.960 \Rightarrow m = \gamma a_w = 16$

Elbert's magnetostrophic coexistence range (MG_2) - Nakagawa's 3rd Case

$\Lambda = 0.72, \mathrm{Ch} = 6.0 \times 10^3, \mathrm{Ra} = 1 \times 10^6$



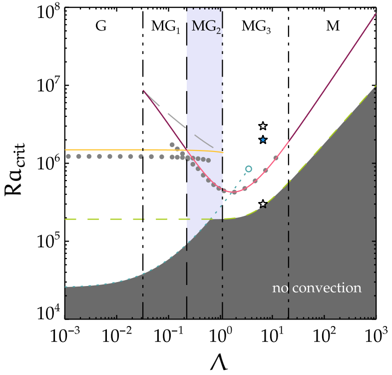


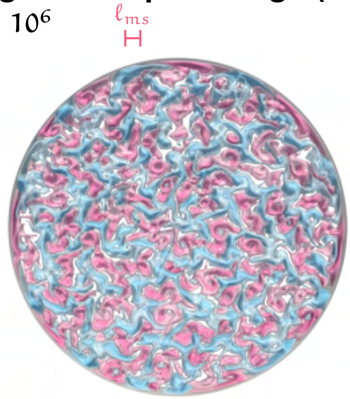
vertical velocity u_z, topview

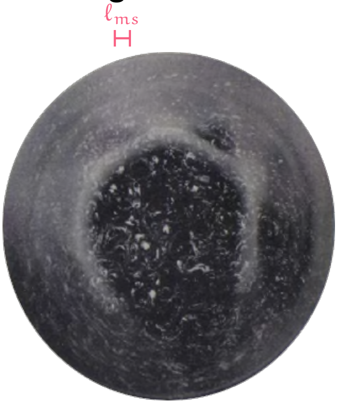
- stationary magnetostrophic modes: $Ra/Ra_{ms} = 1.8$, $a_{ms} = 3.512 \Rightarrow n \approx 9$
- oscillatory: $Ra/Ra_O = 2.4$, $a_O = 11.501 \Rightarrow n \approx 27$
- wallmodes: $Ra/Ra_w = 1.6$, $a_w = 3.938 \Rightarrow m = \gamma a_w = 16$

Magnetically dominated magnetostrophic range (MG_3) - Nakagawa's 4th Case

$\Lambda = 6.6, \mathrm{Ch} = 5.5 \times 10^4, \mathrm{Ra} = 2 \times 10^6$



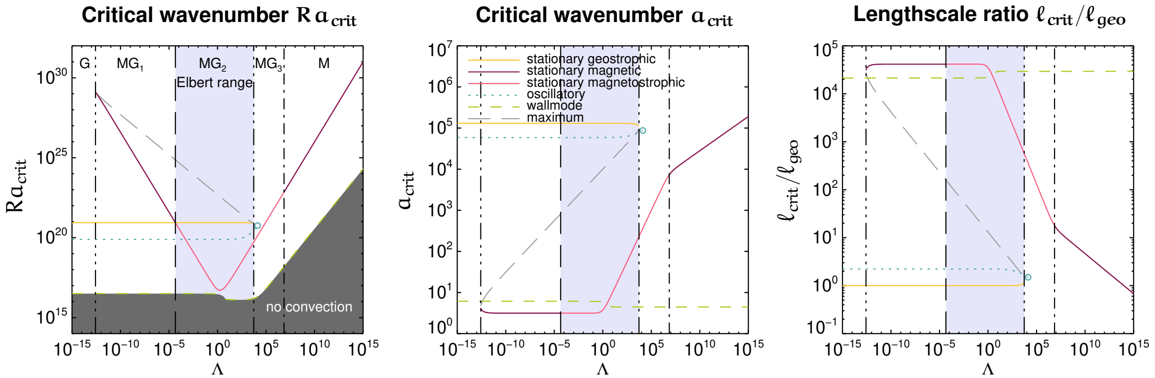




vertical velocity u_z, topview

- no oscillatory modes
- stationary magnetostrophic modes: $\mathrm{Ra}/\mathrm{Ra}_{ms} = 2.7$, $a_{ms} = 7.941 \Rightarrow n \approx 20$
- wallmodes: $\mathrm{Ra}/\mathrm{Ra}_w = 7.1$, $a_w = 4.010 \Rightarrow m = \gamma a_w = 16$

Back to Earth . . .



- ⇝ Elbert range geophysically most relevant
- ⇝ linear analysis suggest boundary-attached, oscillatory, and geostrophic and magnetostrophic stationary modes are excited at Earth-like values
- ⇝ $\sim$ 5 orders of magnitude difference between magnetostophic and geostrophic modes

. . . and Other Planets

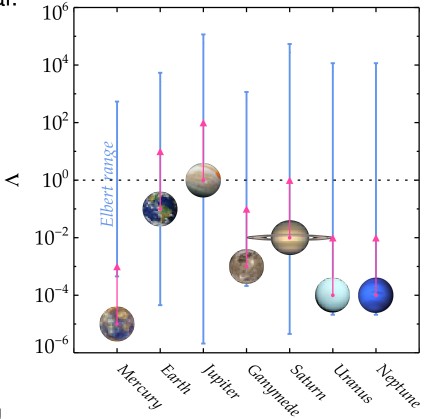
- linear theoretical predictions carry over to nonlinear, turbulent flows
- liquid metal rotating magnetoconvection is strongly multimodal:
  - oscillatory and boundary-attached modes
  - **geostrophic, magnetostrophic, and magnetic stationary modes**

⇝ large-scale magnetostrophic mode appears not dominant

⇝ thermal-inertial oscillatory modes punch above their weight

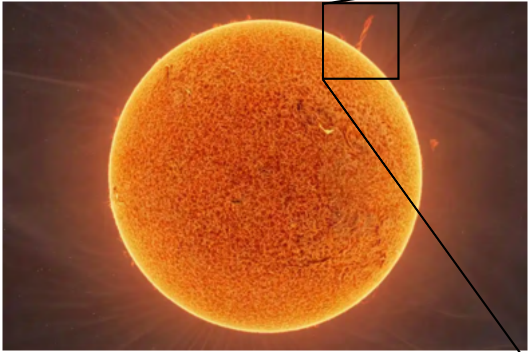
- Elbert Range (MG_2) coincides with planetary estimates

$$\frac{4}{3}(4\pi^2 \mathrm{Ek})^{1/3} < \Lambda < \frac{1}{2}(3^4\pi^2 \mathrm{Ek})^{-1/3}$$



⇝ We need more extreme experiments and DNS of this system!

Back to the Sun - Solar Tornadoes[†]



- ∼ 14 Earths high, lasted 3 days
- may contribute to solar coronal heating*
- or is it Alfvén waves?

*Kuniyoshi et al., *Astrophys. J.* 949 (2023)

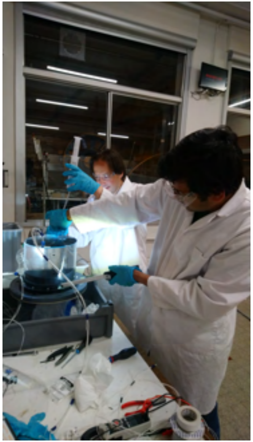
†Image: J. Guenzel, A. McCarthy

Creating Magnetic Tornadoes in the Lab - LEE2

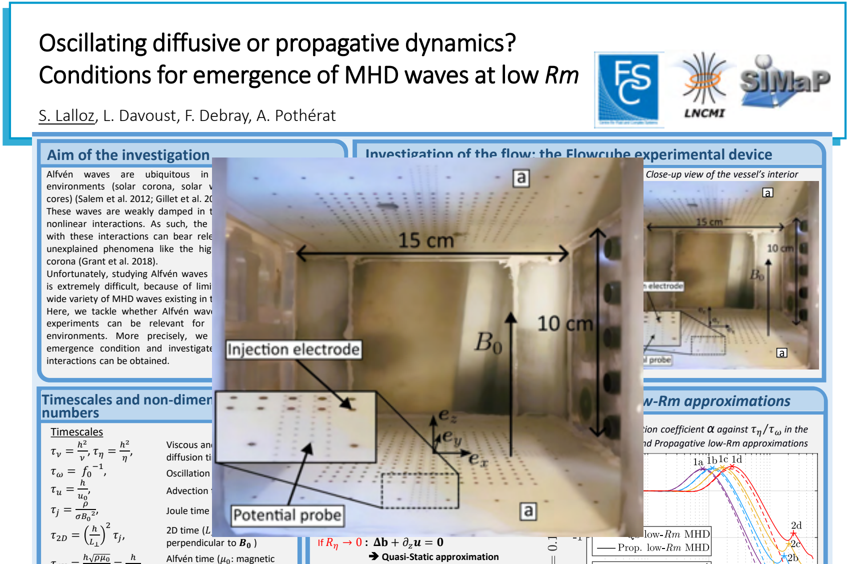
Little Earth Experiment 2 - Sulfuric Acid in a 10 T Magnet

EMFL European Magnetic Field Laboratory
LNCMI

- Centre for Fluid and Complex Systems, operated at LNCMI Grenoble



Alfvén Waves in Liquid Metals - Flowcube



Understanding Tangent Cylinder Physics

Magnetic Taylor-Proudman Constraint explains flows into Tangent Cylinders

Alban Pothérat[1], Kélig Aujogue[1], Rishav Agrawal[2] and François Debray[3]
alban.potherat@coventry.ac.uk
[1]Centre for Fluid and Complex Systems, Coventry University, UK, [2]School of Engineering, University of Liverpool, UK, [3]Laboratoire National des Champs Magnétiques Intentes, CNRS Grenoble, France

Motivation

The flow of
netic field.
any flow th
tation axis.
the TC[6, 7]
controlled l
model of th
forces with

Magne

- Aligned
- Inductio

$Ra = \frac{\alpha g(}{}$

Rewriting t
such that c

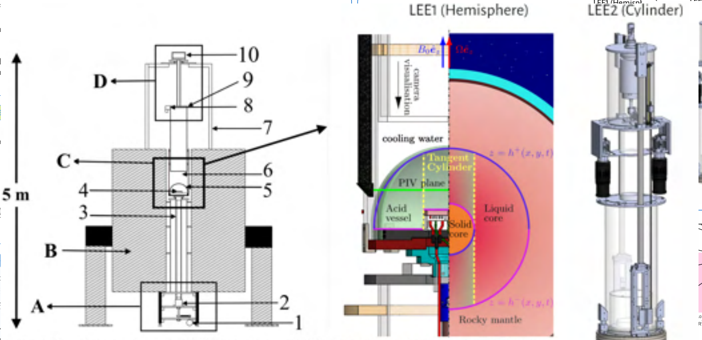
$\nabla_\perp$

Magnetic T
solenoidal
current c_w

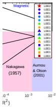
$$\boxed{c_\perp \cdot \nabla_\perp h = \partial_t h + c_w + \mathcal{O}(Ek^{1/2}, Ro)}$$

With static ($\partial_t h = 0$) and insulating ($c_w = 0$) boundaries, the current $c_\perp$ follows surfaces of constant height.

The Little Earth Experiment: LEE1 and LEE2 [3, 5, 9, 1]



Left: Schematic of the complete system inside the magnet. Right: Main vessel. A: Driving module, B: 10T magnet, C: Main Vessel, D: Measurement system. 1: Motor, 2: Rotary Union, 3: Torque Tube, 4: Liquid Heater, 5: Dome, 6: Cooling Water, 7: Supporting structure, 8: PIV Camera, 9: Optical speed sensor, 10: Wirelessly controlled laptop recording data in the rotating frame, 11: Mirror, 12: LASER diode, 13: K-type thermocouples connected under and in the ceramic plate, 14: Pipe carrying the ethylene glycol



Zonal and Radial Flows in LEE1's Elbert Range [5, 9]

Velocity profiles of radial and azimuthal flows for Ek = 1.3 × 10⁻⁵, Ro = 0.045 ± 0.015

Thank you for your attention!